Nonprogressive Diffusion on Social Networks: Approximation and Applications

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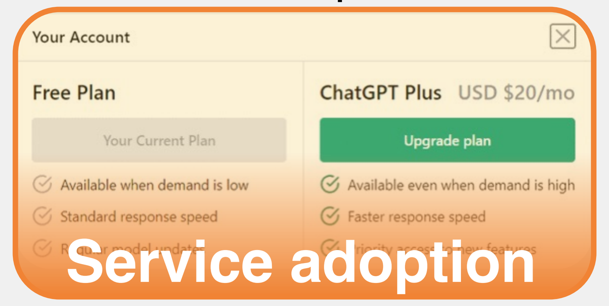
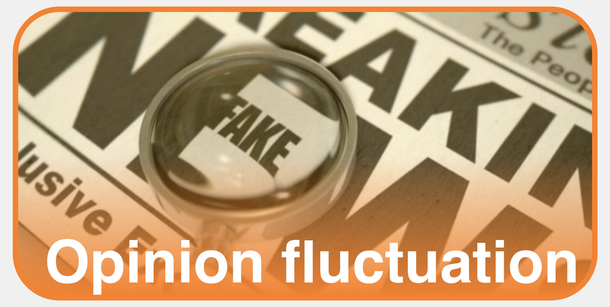
1. Introduction

Nonprogressive diffusion refers to the scenario that an agent's behavior can be influenced by neighbors, causing transitions between the unadopted and adopted statuses, and vice versa.





Challenges of nonprogressive diffusion

- **Evaluation** of diffusion influence — correlation, dynamics, topology
  e.g., the expected adoption rate for each user.
- **Decision-making** in a social network — no closed-form, nonconvexity, large scale
  e.g., the set of seed users, price of the product.

Research Goal: We aim to provide an approximation scheme for optimization problems associated with nonprogressive diffusion with both **theoretical guarantee** and **runtime efficiency**.

2. Diffusion Model

Arbitrary network topology	Arbitrary intrinsic utility
User set V | Neighbor set $\{\mathcal{N}_i\}_{i\in V}$	Fixed preferences $\{v_i\}_{i\in V}$
Not limited to specific structure (e.g., random graph, regular graph)	Not limited to specific distribution (e.g., uniform distribution, normal distribution)

Network effect strength among all neighbors; Adoption status of user j at time $t-1$

$$Y_{i,t} = \mathbb{1}\{u_{i,t} \geq 0\} = \mathbb{1}\left\{v_i + \beta\frac{\sum_{j\in N_i} Y_{j,t-1}}{|N_i|} + \epsilon_{i,t} \geq 0\right\}$$

Intrinsic utility of user i; Set of neighbors of user i; Random utility with mean zero

$Y_{i,t}=0$ corresponds to unadopted; $Y_{i,t}=1$ corresponds to adopted.

3. Fixed-point Approximation

We propose the fixed-point approximation $\boldsymbol{\mu}^*$ to approximate the limiting expected adoption probability $\boldsymbol{q}^*$ for nonprogressive diffusion.

Limiting expected adoption

$$\boldsymbol{q}^* = \lim_{t\to\infty} \mathbb{E}[\mathbf{Y}_t]$$

$$q_{i,t} = \mathbb{E}_{\mathbf{Y}_{t-1}}\left[\Pr\left(\epsilon_{i,t} \geq -v_i - \beta\frac{\sum_{j\in N_i} Y_{j,t-1}}{|N_i|}\right)\right]$$

Fixed-point approximation

$$\boldsymbol{\mu}^* = \lim_{t\to\infty} \boldsymbol{\mu}_t$$

$$\mu_{i,t} = 1 - F_\epsilon\left(-v_i - \beta\frac{\sum_{j\in N_i}\mu_{j,t-1}}{|N_i|}\right)$$

THEOREM

$$\|\boldsymbol{q}^* - \boldsymbol{\mu}^*\|_\infty \leq C\sqrt{1/N_{\min}}$$

minimal degree

Proof (fixed-point sandwich)

Upper bound fixed-point approximation

$$q_i^* = \lim_{t\to\infty}\mathbb{E}[Y_{i,t}]$$

$$\triangleq$$

$$\mu_i = 1 - F_\epsilon\left(-v_i - \beta\frac{\sum_{j\in N_i}\mu_j}{|N_i|}\right), \forall i \in V$$

Lower bound fixed-point approximation

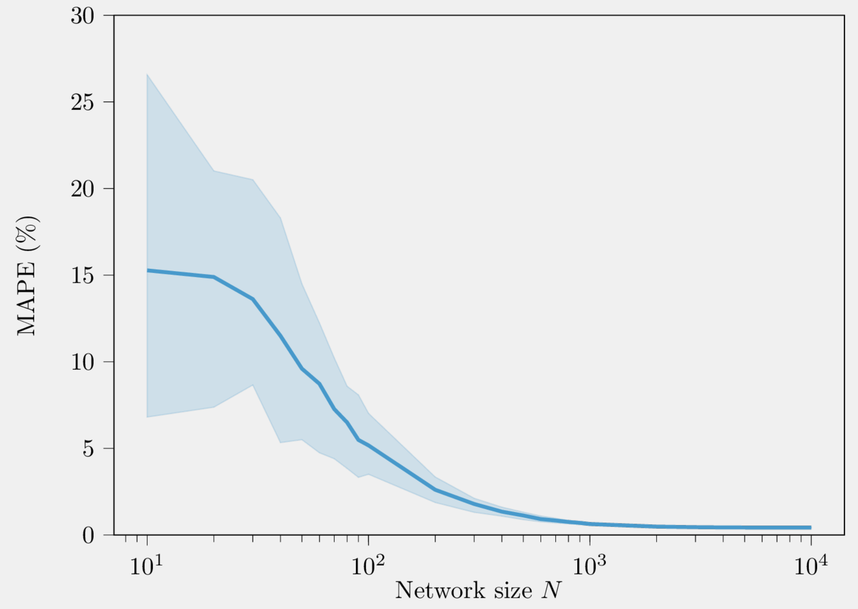
Step 1 Bound covariance

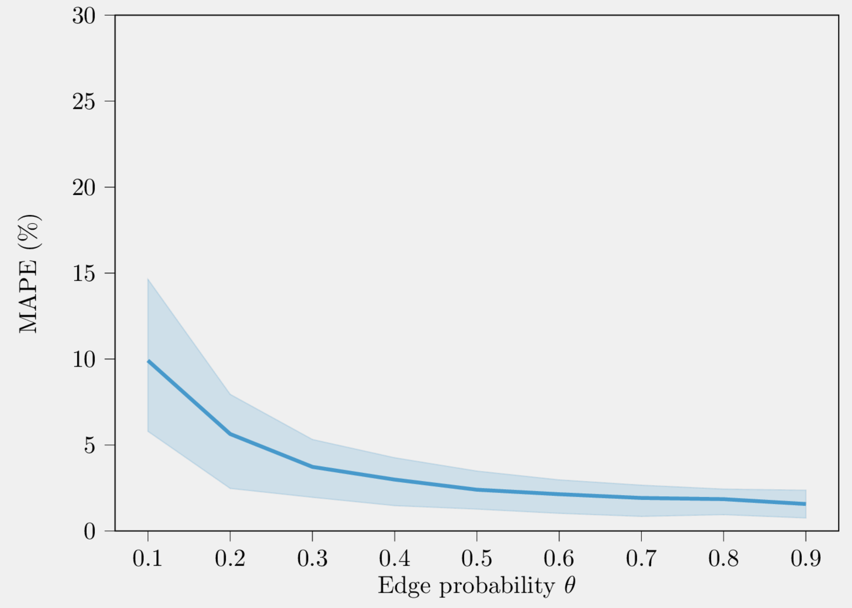
- Topology: between $Y_{i,t}$ and $Y_{j,t}$, $\forall\, i,j \in V$
- Time: between $Y_{i,t}$ and $Y_{i,t+1}$, $\forall\, t \geq 0$
- $\mathrm{cov}(Y_{i,t}, Y_{j,t}) = O(1/N_{min})$

Step 2 Bound dynamics

- Nonlinearity: between $Y_{i,t}$ and $Y_{i,t+1}$, $\forall\, t \geq 0$
- Construct fixed-point approximation by adding (subtracting) a term related to $\mathrm{cov}(Y_{i,t}, Y_{j,t})$.

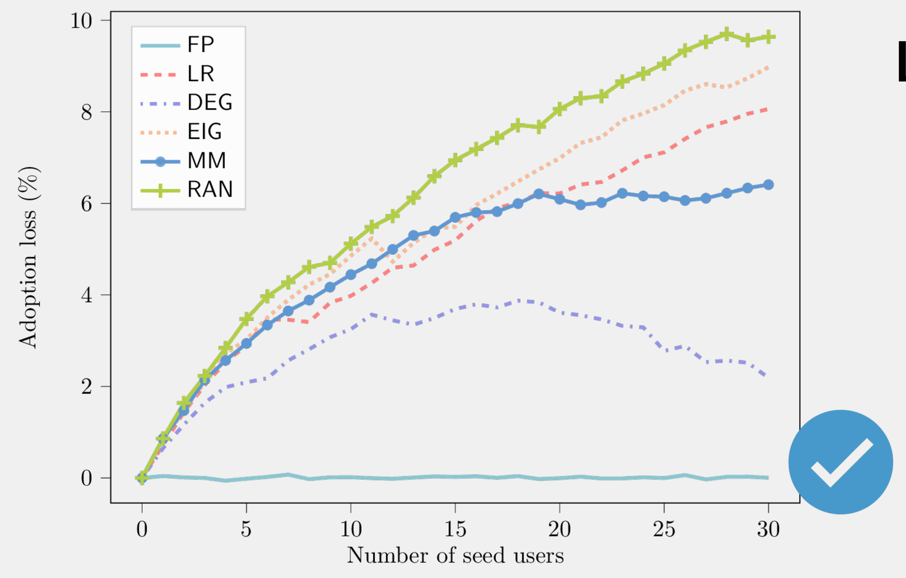
Smaller approximation error can be achieved when the network is cumbersome to simulate (e.g., large and/or dense network).





4. Applications

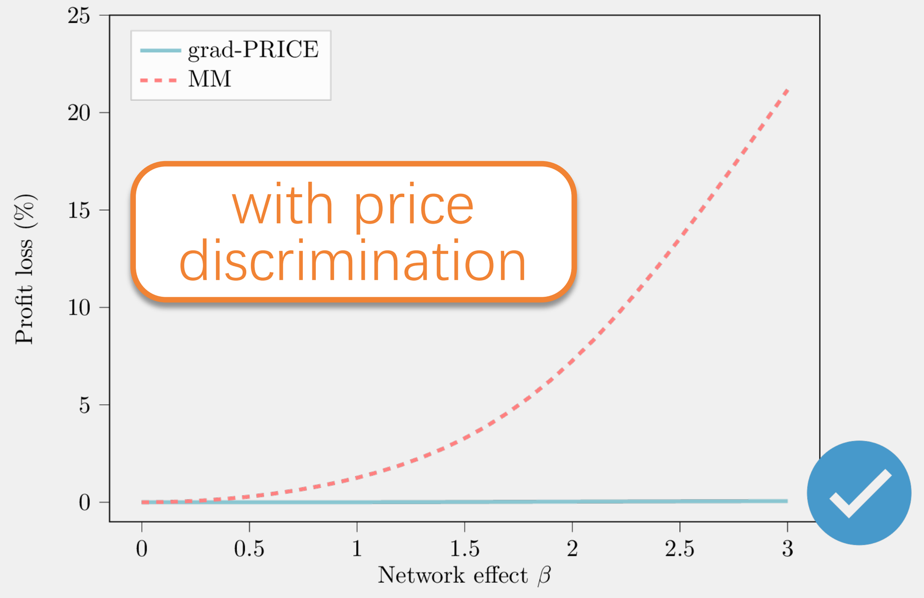
Influence Maximization Problem — Preserve submodularity

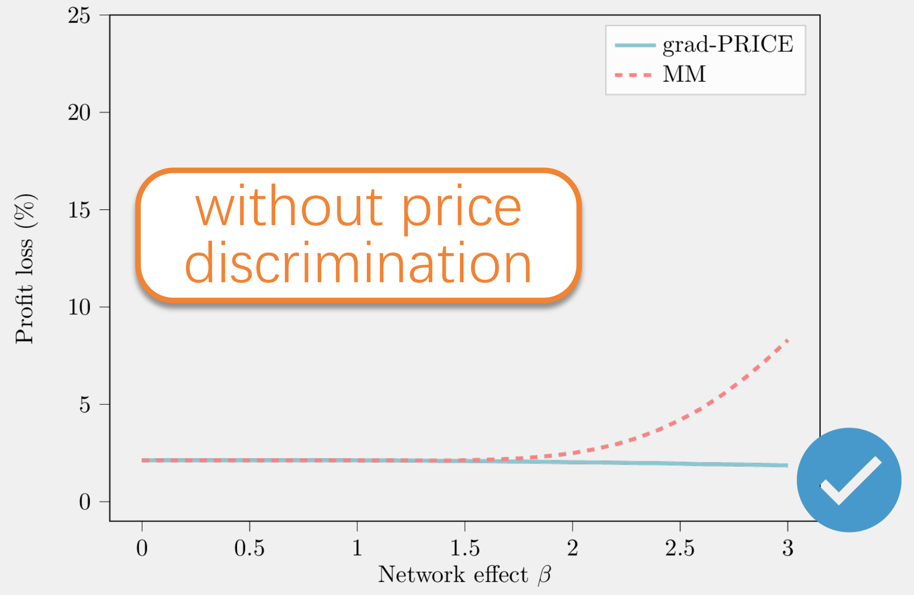


Loss is almost 0 for different scenarios and outperforms the benchmarks

- FP: fixed-point approximation
- LR: simulation with low resolution
- DEG: largest degree
- EIG: largest eigenvector centrality
- MM: model misspecification without network
- RAN: random selection

Pricing Problem on a Social Network — Gradient method applicable





Loss is almost 0 for different scenarios and outperforms the benchmark

5. Conclusion

In this study, we focus on nonprogressive diffusion in the social network.

- We build a general **nonprogressive diffusion model** that is agent-based, considers the local network effect, and can be adapted to many utility models.
- We propose a **fixed-point approximation** that can accurately and efficiently approximate the limiting adoption probability for all agents under the diffusion model.
- Based on the approximation, we investigate the conventional optimization problems and achieve an acceleration of **~1,000 times** with only a slight compromise in accuracy.